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Statistical properties of corporate board and director networks

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Abstract. The boards of directors of the largest corporations of a country together with the directors form a dense bipartite network. The board network consists of boards connected through common directors. The director network is obtained taking the directors as nodes, and a membership in the same board as a link. These networks are involved in the decision making processes relevant to the macro-economy of a country. We present an extensive and comparative analysis of the statistical properties of the board network and the director network for the first 1000 US corporations ranked by revenue ("Fortune 1000") in the year 1999 and for the corporations of the Italian Stock Market. We find several common statistical properties across the data sets, despite the fact that they refer to different years and countries. This suggests an underlying universal formation mechanism which is not captured in a satisfactory way by the existent network models. In particular we find that all the considered networks are Small Worlds, assortative, highly clustered and dominated by a giant component. Several other properties are examined. The presence of a lobby in a board, a feature relevant to decision making dynamics, turns out to be a macroscopic phenomenon in all the data sets.

PACS. 87.23.Ge Dynamics of social systems – 89.65.-s Social and economic systems – 89.65.Gh Economics, econophysics, financial markets, business and management

1 Introduction

The boards of directors of the corporations of a country form together with the directors a bipartite network. The board network consists of boards connected through common directors. The director network is the network obtained taking the directors as nodes, and a membership in the same board as a link. It is well known that the director network of the largest companies in the US and in other countries has a high degree of interlock, meaning the fact that some directors serve on several boards at the same time so that many boards are connected by shared directors. Interlock conveys information and power (i.e. banks lending money to a firm can use interlocked directors in firms of the same industrial sector to get information about the real risk of the loan). In Figure 1 the network of boards with two or more shared directors in the Italian Stock Market is represented.

It has been argued that in a capitalistic economy, as a consequence of economic power concentration, "a special social type emerges spontaneously, a cohesive group of multiple directors tied together by shared background, friendship networks, and economic interest, who sit on bank boards as representative of capital in general" [1]. Now, while part of the public opinion has been since long ago concerned about the fact that the corporate *élite* would represent a sort of "financial oligarchy controlling

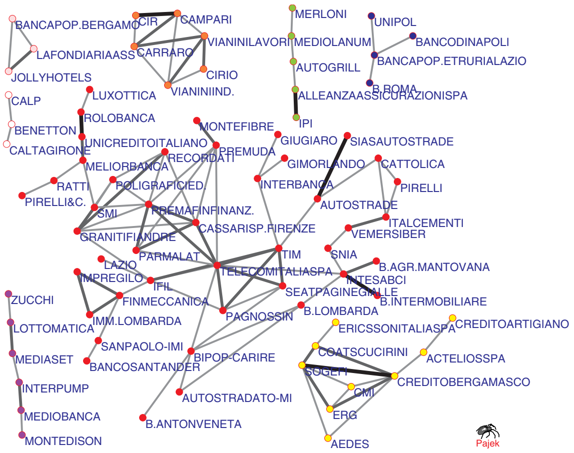

Fig. 1. The Italian Stock Market: the network of boards with two or more shared directors. Gray, dark gray and black links correspond to 2, 3, and 4 shared directors respectively.

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the business of the country" [2], stockholders are more concerned about the effectiveness of boards in overseeing management. Board's directors should in fact monitor managers's strategies and decisions to the interest of stockholders. Recently, after several cases of bankruptcy in the western countries (Enron, Vivendi, Parmalat in 2002–2003), the role of boards in the decision making process is under examination and more sophisticated forms of corporate control are often advocated by the public opinion.

Two issues raise very naturally about directors interlock networks: the first is the characterization of the topological properties of the board network and the director network. The second issue stems from the fact that large corporations leading the economy of a country have their boards organized in a network: one can ask if and how the structure of these networks influences the decision making process in which directors are involved.

Davis and collaborators have shown [3] that the director network and the board network of the Fortune 1000 corporations has Small World properties in the sense of Watts and Strogatz [5]. Newman, Watts and Strogatz [4] have applied on the same data set a generalized random graph model showing that using the generating function method, it is possible to reproduce very accurately the degree distribution of the director network. On the contrary, their model fails in predicting the degree distribution of the board network. In fact the director network turns out to be *assortative*, as observed commonly in social networks, meaning that directors with high (low) degree tend to be connected to directors with high (low) degree. As a consequence, even if the random graph model predicts the right degree distribution for the director network, it underestimates the number of boards with high number of interlocks and with small number of interlocks.

As a general empirical finding, social networks are characterized by assortativity [6] and high *clustering coefficient* $\bar{c}$ (the latter measuring the average fraction of connections between the first neighbors of a node out of all the possible connections among them). Catanzaro et al. [7] investigate a formation network mechanism that generate assortativity in networks. In such a model though the clustering coefficient of nodes of degree k is not decreasing with k as observed in real networks. Newman and Park [8] have recently argued that the presence of groups or communities in a social network is able to produce alone both assortativity and clustering. They develop a model in which nodes belong to one or more groups and have probability p to be connected to another node of the same group. Instead they are never connected to nodes of groups they do not belong to. If groups have heterogeneous size, than nodes who belong to a small group tend to have low degree and are connected to others in the same group, who also have low degree. This model explains about 40% of the observed assortativity in the Fortune 1000 network. It is worth emphasizing that the model of Newman and Park is *not* a network formation model: directors are assigned at random to boards in such a way that the number of boards per director and the number of directors per board are distributed as in the real data. The model does not consider a local mechanism by which directors are recruited in a board. A possible explanation for the discrepancy in the assortativity is that new board members are more likely to be recruited among those who are already connected to some of the current board members. This would mean that the sociological mechanisms which are at work in shaping the topology of the network can not be neglected.

Some recent works have focused on the influence of the structure of the interlock network on the decisions made by boards. There are essentially two kinds of decisions a board is faced to. *Local* decisions regard topics specific to the board, such as the appointment of a vice president, for which boards can be assumed not to influence each other. Battiston et al. [9] investigate by means of a decision making process model, how a minority of well connected directors can influence significantly the decision of the majority. By contrast, *global* decisions such as for instance whether to increase or decrease investments in development or in advertisement, depend on the belief in economical growth or recession. In these cases, decisions previously made in some boards might influence other boards, through the presence of shared directors. In a recent model, Battiston et al. [10] investigate the conditions under which a large majority of boards making a same decision can emerge in the network. Similar issues concern of course not only the boards of large corporations, but also many governance structures in social institutions.

In this paper we report an extensive and comparative analysis of the topological properties of the board network and those of the director network from three data sets: the corporations of Fortune 1000 for the year 1999 and the companies quoted in the Milan Stock Exchange Market for the years 1986 and 2002. All the considered networks are Small World networks, assortative and highly clustered. They all have a highly connected giant component. The presence of a *lobby* in a board turns out to be a macroscopic phenomenon in all data sets.

2 Data analysis

The data sets we consider span over different countries and over time. We analyzed the composition of the boards of the Fortune 1000 corporations in 1999 (1000 companies) and the boards of the quoted companies in the Milan Stock Market in two temporal snapshots: 1986 (220 companies) and 2002 (240 companies). Data have been collected from technical publications used by stock market operators [11,12]. The 1986 data include the so called *restricted market*, i.e. the market of companies quoted only in certain cities (for example, only Milan or Rome, and not in the whole national market). This slice of the market is absent in 2002 data.

2.1 The bipartite graph structure

A bipartite graph consists of two separate classes of nodes, while an edge always connects a node of one class to a node

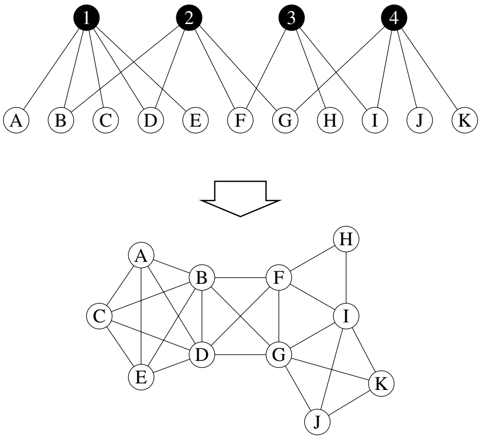

Fig. 2. A bipartite graph and its one-mode projection. Nodes labelled by numbers corresponds to boards, nodes labelled by letters corresponds to directors. After [4].

of the other one. An example is reported in Figure 2. A node represents alternatively a director or a company. A link between two nodes represents the fact that the director sits in the board. We have an *interlock* when a director serves on the boards of two companies. If two directors of a given board, serve together as well in another board, we then have a *multiple interlock*. We call *lobby* the subset of directors of a board who serve on an outside board together with a director of the present board (after [9]). In fact, the members of such a sub-group will have stronger connections among each other than with the other members of the board, and they will have common interests outside the company under consideration.

As it is well known, the bipartite graph can be projected into two one-mode networks. In the projections two boards (directors) are connected if they have at least one director (board) in common. The projections will be referred to as the *director network* and the *board network* (for example Fig. 1). In the operation of projection some information is lost: consider for instance three directors connected in a triangle. The links do not specify whether each pair of directors sit in a different board or whether the three directors sit all in the same board. Without interlocks the director network would split up into disconnected clusters each of which completely connected. Clusters would correspond in this case to the boards. The two networks have *weighted edges*: two boards can share one or more directors and two directors can co-serve in one or more boards.

A bipartite graph can be represented in a compact way by the adjacency matrix:

$$C_{\alpha i} = \begin{cases} 1 \text{ if } \alpha \text{ sits in board } i \\ 0 \text{ otherwise.} \end{cases} \tag{1}$$

This is an $M \times N$ matrix, M being the number of directors, and N being the number of companies. This is a binary matrix, and in general it is neither square, nor symmetric. For the one-mode projection relative to the boards, we should take into account that the number of directors sitting in boards i and j, is equivalent to the number of paths of length 2 connecting i and j in the bipartite graph. Therefore, this number, that gives the weight of the connection between i and j, can be expressed in terms of the adjacency matrix. In the end, defining the adjacency matrix of the projection as

$$B_{ij} = \begin{cases} w_{ij} \text{ if } i \text{ and } j \text{ are connected with weight } w_{ij} \\ 0 \text{ if } i \text{ and } j \text{ are not connected} \end{cases} \tag{2}$$

the entries are:

$$B_{ij} = \sum_{\alpha} C_{\alpha i} C_{\alpha j}. \tag{3}$$

In terms of matrix product:

$$B = C^T C. \tag{4}$$

In analogous way:

$$D_{\alpha\beta} = \sum_{i} C_{\alpha i} C_{\beta i}. \tag{5}$$

And,

$$D = C C^T. \tag{6}$$

While the off-diagonal entries correspond to the edges weights, the *diagonal entries*, are, respectively, the **size** B_{ii} of *board i*, and the number $D_{\alpha\alpha}$ of boards a director α serves on.

2.2 Structure and average quantities

In Table 1 we report some average quantities concerning the two networks. For sake of comparison, we report in the same table the values concerning other two networks that have been well studied in recent years. The first one, “cond-mat”, is the network of authors and articles in condensed matter physics, archived at Los Alamos Laboratories [14,15]. The second one, “A.S. Internet 1999”, is the Internet map, as it appeared in 1999, considered at the autonomous systems level [16]. While the first is a social network and we expect to observe some similarity with the the director network under study, the second one is a technological network.

From a global point of view, we first notice that the fraction N_c/N of nodes belonging to the maximal connected component is larger than 0.8 for all the networks. If one selects only the links above a certain threshold of weight, the network starts to split in several connected components of comparable size (Fig. 1). One can try to individuate groups of interest in these connected components. However, two companies can belong to the same group of interest and yet do not share even one single director. So a cluster analysis on the board network should be complemented with a cluster analysis of the economical ties among companies, such as the ownership relationships. On the other hand, in term of graph analysis, ownership networks in the stock markets are quite peculiar and specific quantities need to be introduced to replace efficiently the notion of in-degree and out-degree,

Table 1. Average and global quantities for board network (marked with B), the director network (marked with D), cond-mat and Internet. N = number of nodes, E = number of edges, N_c/N = fraction of nodes belonging to the maximal connected component, $\langle k\rangle/k_c$ = average degree over $N-1$, b = average site betweenness, $\bar{c}$ = average clustering coefficient, d = average distance.

	$B,86$	$B,02$	B,US	$D,86$	$D,02$	D,US	$C-M$	$AS,99$
N	221	240	916	2378	1906	7680	16725	5287
E	1295	636	3321	23603	12815	55437	47594	10100
N_c/N	0.97	0.82	0.87	0.92	0.84	0.89	0.83	–
$\langle k\rangle/k_c(\%)$	5.29	2.22	1.57	0.84	0.71	0.79	0.03	0.07
b/N	0.736	0.875	1.080	1.116	1.206	1.384	1.932	2.21
$\bar{C}$	0.356	0.318	0.376	0.899	0.915	0.884	0.327	0.241
d	3.6	4.4	4.6	2.7	3.6	3.7	6.4	3.7

as recently shown by Battiston et al. [17]. Therefore the connected components obtained by setting a threshold on the links weights carry only a partial information. A more thorough investigation of this aspect will be carried out in future analysis.

Another interesting aspect, is that both the board and the director network are much less sparse than the comparison networks. A measure of the sparsity of a network is given by the ratio of its average degree $\langle k\rangle$ and the degree k_c that each node would have if the network were completely connected ($k_c = N - 1$, where N is the number of nodes). The value of $\langle k\rangle/k_c$ for the board network is larger than the one for the director network, which is anyway one order of magnitude higher than the one of cond-mat and Internet.

Finally, the network displays small-world property. The average distance between two nodes of the maximal connected component is always of the order of a few units, thus of the order of $\log(N)$, N being the total number of nodes.

The clustering coefficient is around 0.9 in the director network and around 0.35 in the board network. But to what extent the clustering is simply due to the fact that directors are organized in groups (the boards)? It is worth noticing here the two-sided effect of the interlock on the clustering coefficient. In the absence of interlock, directors would be connected to all the other directors of their board and to no other directors outside their board. Therefore the clustering would be 1 for the director network and 0 for the board network. If now a director of board i serves also on board j, then his clustering coefficient will be much less than 1 because his neighbors in board i and j are not nearest neighbors among each other. Thus the interlock decreases the clustering coefficient of the director network and increases the clustering of the board network. We didn't find in the literature an estimation of the clustering coefficient that would be observed in these networks if they had the same connectivity distribution but edges were drawn at random, in analogous way as it has been done by Newman and Park [8] for the assortativity. Therefore we cannot say to what extent the observed clustering is due to sociological mechanism shaping the evolution of the network. This will be the subject of a future work.

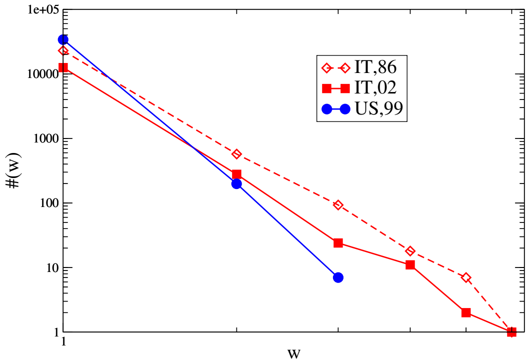

Fig. 3. Director network: distribution of edges weights, i.e. the number of boards on which two directors co-serve. The range of the horizontal axes w is [1 6]. A difference is visible between the American and the Italian market in both the slope and the range.

2.3 Distributions

Let us now move from the average quantities toward the statistical distributions. In Figures 3, 4, the weight distributions are displayed. Weights are the off-diagonal elements of the adjacency matrices and correspond respectively to the number of boards on which two directors co-serve and the number of directors shared by two boards. The range is too small to reasonably fit the distribution. What is clear is that the Italian network show a lower slope, as well as a higher maximum weight than those displayed by the US network (two boards with six common directors and two directors with eight common companies). The distribution of the number of chairs, namely the number of boards (Fig. 5) on which each single director serves, shows that most of the directors serve only on one board and that directors serving on several boards are less and less probable. There are of course more chairs than directors, but the ratio of the number of available chairs over the number of directors in the three data sets is 1.2297 (US), 1.2769 (IT '02), 1.3646 (IT '86). Thus in principle, chairs could be assigned in a way so that no director holds more than two chairs. If instead the exceeding chairs were assigned at random, then the distribution of the number of chairs per director would follow a Poisson

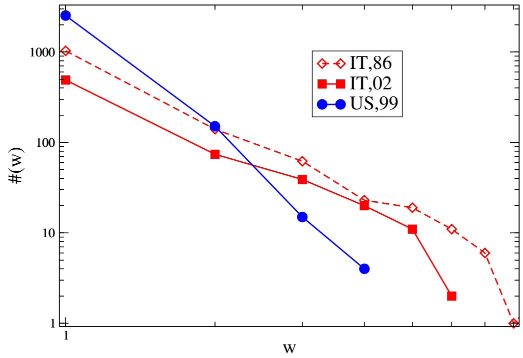

Fig. 4. Board network: distribution of edges weights, i.e. the number of directors shared by two boards. The range of the horizontal axes w is [1 8]. A difference is visible between the American and the Italian market in both the slope and the maximum w value.

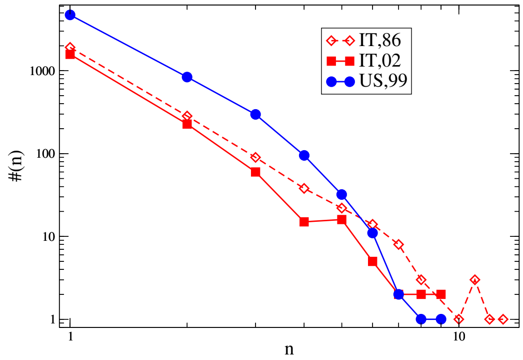

Fig. 5. Distribution of the number of boards on which a single director serves.

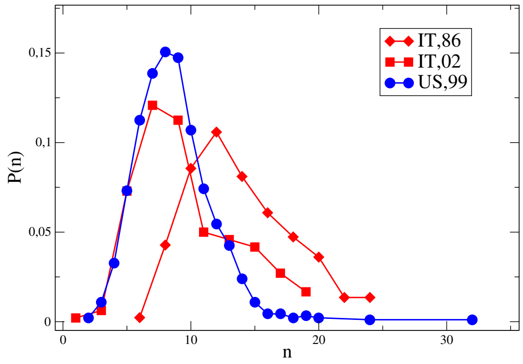

Fig. 6. Distribution of the size of the boards (the number of directors sitting in a single board).

distribution. In all data sets, starting from 5 chairs the probability of holding m chairs deviates from the Poisson distribution by one or several order of magnitude, meaning that directors with more than 5 chairs are far from being purely random events. However, the range is too small to try to fit the functional form of the observed distribution. The distribution of the size of the boards (Fig. 6). displays a characteristic scale around the number of 10 members, and a maximum size around the number of 30.

The cumulated degree distribution of the director network (Fig. 7) displays a plateau up to a degree of 10 as a consequence of the fact that 10 is the characteristic size of a board. Whenever a director has more than 10 links this is essentially the result of holding several chairs. If the number of chairs per director followed a Poisson distribution, then directors with very high degree would be less numerous. In a future work we will investigate in detail which quantities deviate from their counterparts obtained assigning chairs at random. The degree distribution of the board network (Fig. 8) doesn't display any plateau.

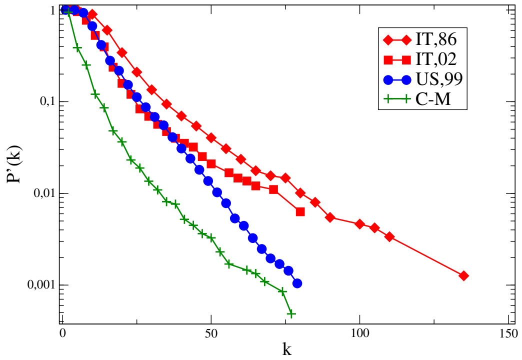

Fig. 7. Director network: degree distribution.

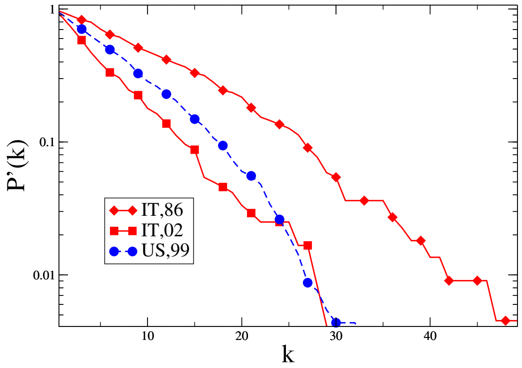

Fig. 8. Board network: degree distribution.

The site-betweenness distribution is displayed in (Figs. 9 and 10). The trend seems quite close to an exponential decay and it is similar across data sets. The site-betweenness measures to which extent nodes are crucial in connecting the different parts of a network. It gives additional information with respect to the connectivity degree. For instance, a node connecting different subnetworks can have large site betweenness but small connectivity degree. A positive correlation between site-betweenness and connectivity degree is displayed in Figures 11 and 12, where the trends can be interpolated with increasing power-laws with slopes 2.2 for the directors and 1.5 for the boards.

2.4 Degree degree correlation and clustering as a function of the degree

As observed in many recent studies, social networks typically display positive degree-degree correlations, also called degree assortativity: nodes tend to connect with nodes of similar degree. This tendency can be measured by means of the assortativity coefficient [6], but a

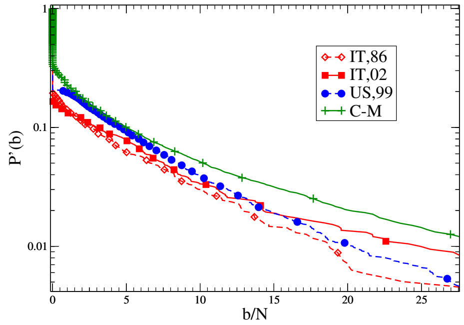

Fig. 9. Director network: site betweenness distribution.

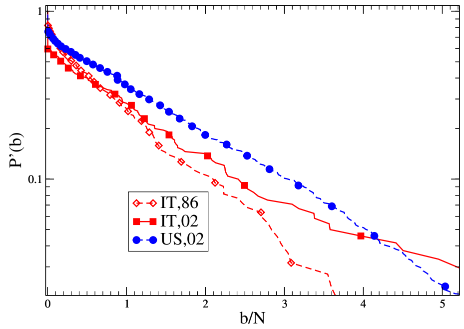

Fig. 10. Board network: site betweenness distribution.

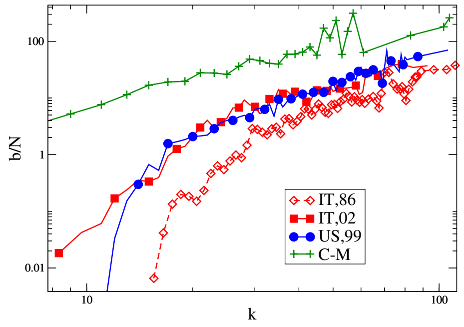

Fig. 11. Director networks: correlation between site betweenness and connectivity degree.

more informative quantity is the average nearest neighbor degree $K_{nn}(k)$ of a node of degree k introduced by Pastor-Satorras et al. [18]:

$$K_{nn}(k) = \sum_{k'} k' P(k'|k) \qquad (7)$$

where $P(k'|k)$ gives the probability that a nearest neighbor of a node of degree k has degree k'. This distribution is increasing, flat or decreasing if, respectively, the network is assortative, un-assortative or disassortative (terms corresponding to positive, null or negative degree correlation). The plots of $K_{nn}(k)$ for the networks under study display a slight increase (Figs. 13 and 14), but the values of the assortativity coefficient, are definitely positive (see Tab. 2). Again both the curves of $K_{nn}(k)$ and the values of r for the Italian '02 and the US '99 data sets are quite close. The Italian '86 data set deviates instead from the others.

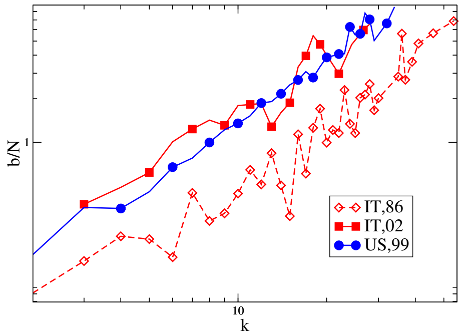

Fig. 12. Board networks: correlation between site betweenness and connectivity degree.

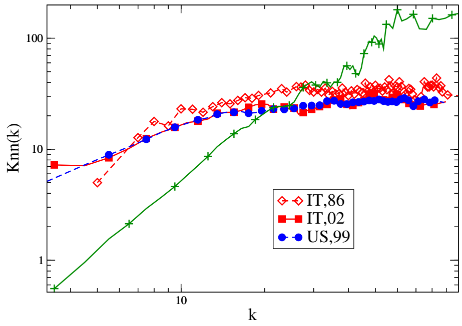

Fig. 13. Director network: average nearest neighbor degree of the nodes of degree k.

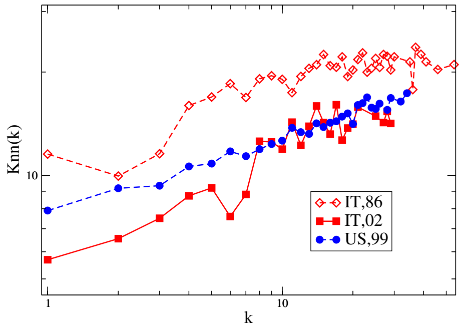

Fig. 14. Board network: average nearest neighbor degree of the nodes of degree k.

Table 2. Assortativity coefficients.

	$B,86$	$B,02$	B,US	$D,86$	$D,02$	D,US
r	0.12	0.32	0.27	0.13	0.25	0.27

A similar scenario occurs for the curves of the clustering coefficient as a function of the degree k of the nodes in the director network (Fig. 16), while for the board network the three data sets have each a different behavior (Fig. 15). In the Italian boards the clustering is very slowly decreasing, while in the US it seems compatible with the trend $c(k) \sim 1/k$ usually observed in social networks, including the director networks shown here. Interestingly this is the only property among the ones analyzed so far for which the two recent data sets (Italy '02 and US '99) deviate significantly from one another.

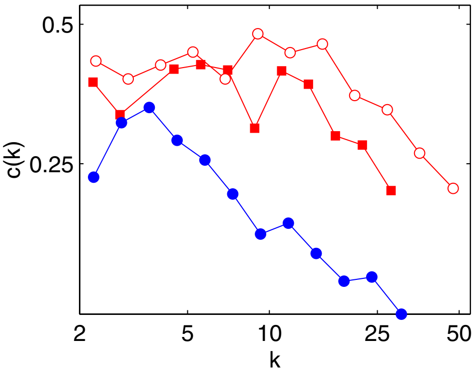

Fig. 15. Board network: clustering coefficient as function of the degree k of the nodes. Colors and markers of the plot as in the previous figures.

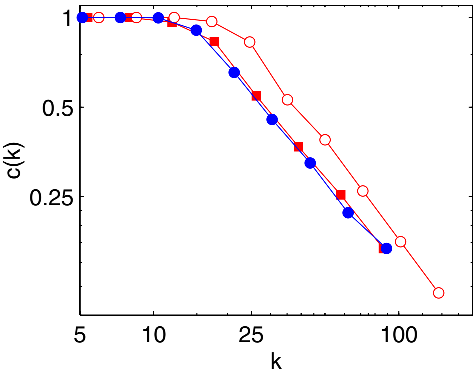

Fig. 16. Director network: clustering coefficient as function of the degree k of the nodes. Colors and markers of the plot as in the previous figures.

3 Conclusions: the emerging picture of the corporate control network

Before summarizing our main results, one further quantity needs to be added to give the magnitude of the multiple interlock. We call lobby the subset of directors of a boards who co-serve on an outside board together with a director of the present board. Thus for instance a lobby of size 2 in board i consists of two directors of the board i who serve as well on board j. We have computed the percentage of boards containing a lobby of size at least 2. It turns out that 35% of US companies and 44% (1986) and 63% (2002) of Italian companies have a lobby of size at least 2, revealing that lobbies are a macroscopical phenomenon. Lobbies are then much more frequent in the Italian data set and this might be related to the slower decay of the clustering as function of the degree. It must be said the the Italian data set includes about 1/4 the number of companies included in the US data set. Moreover the Italian data set includes only quoted companies while the US data set include all kinds of companies ranked by revenue and it is thus a more heterogeneous data set. Battiston et al. [9] have shown that the presence of a lobby can affect the decision making process of boards, allowing a minority to drive the board decision against the interest of the majority. This is then a prominent topic in board network analysis.

In conclusion, from the analysis we have performed, a number of global features seem to characterize the network of the subjects in charge of overseeing the major corporations of a country: all the considered networks are Small World networks, assortative and highly clustered. They all have a giant maximal connected component. Directors with more than 5 chairs seem not to be a random event. We will investigate in the future whether and to what extent the distributions of the quantities presented in this paper deviate from those observed in a network obtained assigning chairs randomly to the directors.

The Italian 2002 data set and the US 1999 data set always display behaviors very similar to each other, with the interesting exception of the curve $c(k)$ of clustering coefficient and the fraction of boards containing a lobby of at least 2 directors. The older 1986 Italian data set deviates almost systematically from the other two more recent data sets, but follows approximately the same type of behavior. The comparison of the quantities we have measured is somehow only qualitative because in most cases these systems do not exhibit neat power law decays nor exponential decays. On the other hand, the common behaviors found in data sets that one could expect very different is remarkable. Now, properties like the clustering and the assortativity are partly due to the organization of the network of directors in groups (the boards). In a future work we will investigate how the quantities analyzed here behave on board and director networks in which nodes preserve their degree but links are rewired randomly. Such analysis will reveal to what extent the structures we observe are due to social mechanisms of attachment versus being simply due to the constraints of the system (number and size of the boards, number of directors and number of their appointments). Because economies are strongly affected by the decisions of the corporate élite, it is clear that the observed features should be taken into account by models of global economy decision making dynamics.

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